

The official newsletter of the International Society for Bayesian Analysis.

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ISBA is a newly created International Scientific Society with the objective of interfacing between Bayesian Statistical methods, and scientific areas such as Engineering, Chemistry, Physics, Economics, Business, Astronomy, Earth Sciences, Education, Psychology, Government Policy-Making, Medicine, and Sociology.

**RENEWAL REMINDER**

If you are interested in joining ISBA (a newly formed international society which interfaces between Bayesian methods, and a spectrum of scientific areas) then please send your name affiliation, address, and E-mail address together with your annual membership fee of \$25 to:

Prof. Gordon M. Kaufman, Treasurer, ISBA  
MIT School of Management  
Room 53-375  
Massachusetts Institute of Technology  
Cambridge, MA 02139, USA

**ANNOUNCEMENT**

The following are two editorials concerning the establishment of a Bayesian journal. Opposed is Dennis Lindley, in favor is Arnold Zellner.

Against A Bayesian Journal

Suppose that a person is in favor of action A but faces a democratic assembly whose permission is needed before they can act. One procedure is to put A to the vote; if it is rejected, to put it again later (preferably when some democrats opposed to A are absent) and repeat until A is passed. There is a simple theorem in probability to show that, under a wide range of conditions, this procedure will terminate by acceptance with probability one. This is what has

Secretary Michel Mouchart, and a nominating committee, appointed at the San Francisco Meeting are still collecting a slate of nominees for our first democratically elected Board. You will be contacted with a ballot when the Editors receive the slate.

**FROM OUR MAILBAG**

The joint proceedings volume containing papers presented at the First World Meeting of ISBA and at the ASA Section on Bayesian Statistical Science Sessions in San Francisco, August 1993, will be published by the American Statistical Association.

happened in ISBA over A, the creation of a Bayesian journal.

Please allow me to state the case against A, since it was inadequately put in ISBA newsletter #2. Bayesian statistics is not part of statistics, in the way that say multivariate analysis is. It is a way of looking at the whole of statistics. For example, there is a Bayesian approach to multivariate problems. What we, as Bayesians, have to do is to persuade statisticians of the superiority of our viewpoint over others. To create a separate journal would defeat this objective because frequentists would argue that they need not consult it since it is not in their field; just as the multivariate expert will sensibly not read a nonparametric journal. In this way, non-Bayesians

will not be exposed to our ideas and hence will not appreciate them. Acceptance of the Bayesian paradigm by the multivariate expert is likely to come through them seeing that their task can be better performed by that method. To create a separate Bayesian journal would deny us this exposure and delay acceptance of the coherent view. Most editors are tolerant of Bayesian papers of high quality; some are themselves Bayesians. Publication in general journals is practicable.

Journals are created for the good of science, not to publish your friend's papers. The good of Bayesian statistics is not served by the creation of a specialist journal

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#### In Favor of a Bayesian Journal

While attending the excellent Symposium on Exploration of the Informational Aspects of Bayesian Statistics in Japan in December, 1993, Seymour Geisser, Jim Press, Jim Berger, Kunio Tanabe, Dale Poirier, Malay Ghosh, John Geweke, Herman Van Kijk, Hajime Wago, Wolfgang Polasek, Arnold Zellner and others discussed the creation of a Journal of Bayesian Analysis (JBA). Several believed that for ISBA to be viable it would have to provide members with a journal. It was agreed that the journal should be a broad one covering general scientific methodology and foundational, theoretical and applied Bayesian analyses in all areas of science and application. Also, special issues devoted to papers with a common theme will be published under the direction of invited guest editors. The topics covered in the invited papers sessions at the ISBA Meeting in San Francisco and at this Symposium in Japan would be ideal for the journal and special issues. To make the journal effective, it was agreed that there be a large international editorial board.

Many at the meeting in Japan volunteered to serve on the editorial board. Also, Seymour Geisser, who was requested at the ISBA board meeting in San Francisco to look into the journal issue, suggested in alternative name, Journal of Bayesian Sciences (JOBS) rather than JBA. Please write to him (School of Statistics, University of Minnesota, Minneapolis, Minnesota 55455, USA

(GEISSER@UMNSTAT.STAT.UMN.EDU) or to me (Grad. Sch. of Bus. U. of Chicago, Chicago, IL 60637, USA Fax (312)702-0458 E-mail: FAC\_AZELLNER@GSBACD.UCHICAGO.EDU) to express your views and/or willingness to participate. The journal issue will be considered further at the next ISBA meeting in Alicante, Spain, June 10-11, 1994. See you there!

Arnold Zellner  
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#### **INTERNATIONAL SYMPOSIUM ON EXPLORATION OF INFORMATIONAL ASPECTS OF BAYESIAN STATISTICS, held December 19-23, 1993 in Yamanashi, Japan**

The symposium consisted of a series of papers on a variety of topics. It was organized to honor Hirotugu Akaike and the Institute of Statistical Mathematics.

#### ***Models, Prior Information and Bayesian Analysis***

Arnold Zellner, University of Chicago

#### ***Structural Shifts in a State-Space Model with Unknown Number of Join Points***

Hiroki Tsurumi, Rutgers University

#### ***Prior Beliefs About Fit***

Dale J. Poirier, University of Toronto, Canada

#### ***Bayesian Considerations of Spatial Prediction***

Noel A.C. Cressie, Iowa State University

#### ***A Bayesian Approach to the Estimation of Upper Crustal Rock Densities Using Gravity Data***

Yasuaki Murata, Geological Survey of Japan

#### ***Deflection of Anomalous Changes of Ground Water Level Related to Earthquakes***

Norio Matsumoto, Geological Survey of Japan

#### ***Nonregular Asymptotic Bayesian Theory***

Kei Takeuchi, University of Tokyo, Japan

#### ***On the Strategy for the Survival of Bayesian Statistics***

Hirotugu Akaike, Institute of Statistical

Mathematics, Japan

**Reference Distributions in Model Choice**

Jose M. Bernardo, Generalitat Valenciana, Spain

**Learning Curves, Generalization Errors and Model Selection**

Shunichi Amari, University of Tokyo, Japan

**Recent Developments Concerning the Intrinsic Bayes Factor for Model Selection**

James O. Berger, Purdue University

**On the Shape of the Likelihood/Posterior in Cointegration Models**

Herman K. van Dijk

**Modeling Volatile Economic Time Series by Gibbs Sampling**

Wolfgang Polasek, University of Basel, Switzerland

**Bayesian Inference in Reduced Rank Regression Models**

John Geweke, University of Minnesota

**Two Computational Methods of Evaluation of Bayesian Models**

Yoshihiko Ogata, Institute of Statistical Mathematics, Japan

**Geodesic Data Inversion Using a Bayesian Information Criterion**

Mitsuhiro Matsu'ura

**Time Series Analysis of Groundwater Radon Using Stochastic Differential Equations**

Tomoyuki Higuchi, Institute of Statistical Mathematics, Japan and George Igarashi, Hiroshima University, Japan

**Smoothness Priors Multivariate Autoregressive Time Series Modeling**

Will Gersch, University of Hawaii

**Markov Chain Monte Carlo Methods and Their Application to Statistics**

Yukito Iba, Institute of Statistical Mathematics, Japan

**Bayesian Analysis of Time-Varying-Parameter Models: A Gibbs Sampling Approach**

Chung-Ki Min, George Mason University

**Rank Selection, Soft Optimization, and Heuristics**

Yu-Chi Ho, Harvard University

**Bayesian Analysis of Co-Integration**

Hajime Wago, Toyama University, Japan

**Estimating Cofactors of Multivariate Nonstationary Time Series -- A Bayesian Approach**

Sadao Naniwa, Kumamoto University of Commerce, Japan and Makio Ishiguro, Institute of Statistical Mathematics, Japan

**Limit Cycle and Multistep [Prediction in an Exponential Autoregressive Model for Nonlinear Time Series**

Nobuhiko Terui, Yamagata University, Japan

**Simultaneous Estimation of Structural Change and order of an Autogressive Model by Akaike's Predictive Likelihood Approach**

Hideo Kozumi, Kobe University, Japan

**On Extending a Stepwise Bayes Procedure**

Eiichiro Funo, Kantogakuin University, Japan

**Learning and Model Selection**

Tetsuya Takahaki, Institute of Statistical Mathematics, Japan

**A Monte Carlo Filtering and Smoothing Method of Nonlinear Non-Gaussian Time Series Models**

Genshiro Kitagawa, Institute of Statistical Mathematics, Japan

**Bayesian Analysis of Lymphatic Spreading Patterns in Cancer of the Thoracic Esophagus**

Akifumi Yafune, Kitasato Institute and University of Tokyo, Japan

**Many Nuisance Parameters, Inconsistent MLE's and Hierarchical Bayes Solutions**

Malay Ghosh, University of Florida

**Bayesian (or Nonbayesian) Seasonal Adjustment**

Tohru Ozaki, Institute of Statistical Mathematics, Japan

**A Smooth Estimation of the Prior Distribution in a Bayesian Model - An Use of EIC (an Extended Information Criterion)**

Makio Ishiguro, Institute of Statistical Mathematics, Japan and Akifumi Yafune, Kitasato Institute and University of Tokyo, Japan

**The de Finetti Transform**

S. James Press, University of California

**Bayesian Interim Analysis**

Seymour Geisser, University of Minnesota

**New Approaches to Bayesian Inference on Cycles in Time Series**

Michael West, Duke University

**Bayesian Simultaneous Estimation in Factor Analysis Model**

Kazuo Shigemasa, Tokyo Institute of Technology, Japan

**Bayesian Analysis of Seasonal Economic Data**

Yasuhito Yoshizoe, Aoyama Gakuin University, Japan

**Origin of Distributions and their Development**

Tadashi Matsunawa, Institute of Mathematics, Japan

**State-Space Modeling of Switching Time Series**

Fumiyasu Komaki, University of Tokyo, Japan

**Nonstationary Time Series Analysis via Time Varying Coefficient VAR Model**

Xing-Qi Jiang, Asahikawa University, Japan

**How to Cope with Improper Priors and Ill-Conditioned Posterior Likelihoods in Numerically intensive Nonparametric Bayesian Methods**

Kunio Tanabe, Institute of Mathematics, Japan

**The Second Biennial Workshop**

The second biennial workshop "Bayesian Statistics in Science and Technology: Case Studies" was held at Carnegie Mellon University in Pittsburgh, Pennsylvania, USA, 9-11, October 1993. The workshops emphasize substantive scientific applications of Bayesian statistics, allowing 2 1/2 hours for each invited paper so that background material may be adequately explained. Contributed papers are screened to make sure they are mainly concerned with an application, rather than presenting some methodology which is illustrated by an example. The goal, throughout, is to advance the application areas while providing concrete cases that may challenge existing techniques or provoke reflection on the strengths and limitations of current Bayesian statistical practice.

The four invited papers, all on the general theme of biomedical applications, were:

**Accurate Restoration of DNA Sequences**, by G.A. Churchill, Cornell University;

**Elicitation, Monitoring, and Analysis for an AIDS Clinical Trial**, by B.P. Carlin, T.A. Louis, and F.S. Rhame, University of Minnesota;

**A Bayesian Model for Organ Blood Flow Measurement with Radiolabelled Microspheres**, by E.N. Brown and A. Sapirstein, Massachusetts General Hospital and Harvard Medical School; and **Incorporating Prior Information into the Reconstruction of Single Photon**

**Emission Computed Tomography Images**, by V.E. Johnson, J.E. Bowsher, and T. Turkington, Duke University.

There were two invited discussants for each paper, and some provided alternative analyses of the data. The contributed poster session featured fourteen papers in the areas of highway engineering, avalanche forecasting, medicine and health care, finance, fisheries management, and chemistry.

The organizers of the workshop are editing a proceedings volume that will contain the four invited papers with discussions and selected contributed papers. The volume will be published by Springer-Verlag. The proceedings volume from the first Case Studies workshop, held in 1991, was published by Springer this past summer, with the title "Case Studies in Bayesian Statistics" (Lecture Notes in Statistics, Volume 83), edited by C. Gatsonis, J.S. Hodges, R.E. Kass, and N.D. Singpurwalla. This volume features the five invited papers from the first workshop, with prepared discussions, nine contributed papers, and a closing discussion. The volume is intended for classroom use, with an index featuring models and prior distributions and complete data sets for three of the papers.

The October 1993 workshop was held in conjunction with the second Morris H. DeGroot Memorial Lecture in the Department of Statistics at Carnegie Mellon, which was delivered by A. Philip Dawid.

Preparations are underway for the third workshop, to be held in October 1995 at Carnegie-Mellon

University. A formal call for abstracts to be considered as invited papers will go out in late 1994. However, the organizers (Constantine Gatsonis, Jim Hodges, Rob Kass, and Nozer Singpurwalla) would like to hear of ongoing projects that might lead to invited papers as soon as possible. Those with suggestions, and volunteers, should contact us. We are especially interested in mature applications of Bayesian statistics, the only conditions at this point being that the statistician must have been an integral member of the team doing the work, and that Bayesian methods must be used explicitly. For more information, call Rob Kass at 412-268-8723 or send e-mail to [kass@stat.cmu.edu](mailto:kass@stat.cmu.edu).

The organizers gratefully acknowledge the support given the second Case Studies workshop by the National Science Foundation, the Army Research Office, and the National Institutes of Health.

## Probability in Outline

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**ABSTRACT:** Probability theory is outlined according to the Bayesian viewpoint, that it is a theory of logical inference founded on criteria of consistency. No knowledge of probability is assumed, though a small degree of mathematical facility and some familiarity with the calculus of propositions will be helpful. Problems arising in some other probabilistic viewpoints are set out.

More than two centuries after it became quantitative, and despite the pioneering work of Laplace, probability is still a contentious area. I shall present what is called the *objective Bayesian* - usually just called Bayesian view, and state the justification for adopting it over other pictures.

Consider the problem of how to reason about propositions, meaning statements which are either true or false, when we have insufficient information to be certain which. It is then appropriate to consider the notion of *strength of belief*: we believe more strongly that it is raining in India at the height of the monsoon season, than that it is raining in the Gobi desert, even though we are not in contact with on-the-spot observers who can give us weather reports. This idea of strength of belief is useful to us because we have

the capacity for belief. Please note that I have not yet used the word 'probability' or made any presumption about it.

We now consider whether we can construct a theory of how strongly we should believe that a proposition is true, given what we know about it: a quantitative theory. You might think that anything to do with belief must be so hazy as to be unquantifiable, because anyone can believe anything, to any degree. *But not consistently*: as soon as the 'conditioning' propositions - that we are in a desert, say - are given, assignment of strength of belief in rain takes place by a well-defined set of rules, yielding the same result whether implemented by a human brain or an electronic computer. We shall shortly set up those rules from consistency requirements. (Of course, the computer does not 'believe' in anything because it does not have that capacity; but it can calculate the number perfectly well.) Confusion arises because different individuals often possess different conditioning information, and so use these rules to assign a different strength of belief to the event. For example, someone else in the desert might have heard a storm warning broadcast, and believe more strongly in rain. But the same rules of reasoning are in use, and we recognize intuitively that this a sensible revision to make.

There are rules of assigning strengths of belief, and rules of manipulating them, so as to incorporate further information, for example. The assignment principle is maximum entropy, to which we shall return. The manipulatory rules, denoting the strength of belief by  $p$ , are known as the product rule,

$$p(AB|C) = p(A|BC)p(B|C),$$

and the sum rule,

$$p(A|C) + p(\bar{A}|C) = 1.$$

(Though I am borrowing the notation of probability theory, I have still made no statements about probability itself.) Since strength of belief is not a physical quantity, the rules which it obeys cannot be field-tested in the same way as Newton's laws. Rather it is a theory of logic, and accordingly it must be internally consistent in its construction. This condition of consistency suffices, happily, to give the product and sum rules. Suppose that we do not know the product rule and write  $p(AB|C)$  as an unknown function of the probabilities  $P(A|BC)$  and  $p(B|C)$  alone. (No other inequivalent combination makes any sense.) This relation is

then applied twice, to decompose the belief-strength of the logical product of three propositions into belief-strengths of single propositions. The decomposition can be done in two different ways but, since the logical product is associative, the results must be the same. This condition sets up a functional equation for our unknown function whose solution is the product rule. The sum rule is correspondingly derived by exploiting associativity of the logical sum of propositions; the logical sum is related to negation by the logical relation  $\bar{A} + A = \bar{A}$ . In fact, if we associate a number with every conditional proposition, then the sum and product rules are just the numerical calculus corresponding to the Boolean calculus of the propositions. The sum and product rules are derived in this manner by the physicist R.T. Cox in 1946.

A theory of belief-strength is precisely what is needed in order to solve the problems with which 'probability theory' is concerned. For example, in estimating the value of a physical parameter from noisy measurements, we want to know how strongly to believe that the parameter takes certain values. The theory tells us this.

People who call themselves "objective Bayesians" take belief-strength and probability to be the same thing, and use the terms interchangeably. *This is because statements* like "From what I know, I strongly believe it will rain" and "From what I know, I say it will very probably rain" mean the same thing to most people. Objective Bayesians are satisfied with the idea that the more strongly you believe in something, the greater is the probability you will give it. Even if you start by accepting only a vague intuitive connection between strength of belief and probability, Cox's consistency argument forces these both to obey the same rules, known as 'the law of probability' and given above. Consequently it doesn't matter if you prefer a different term - such as propensity or likeliness or anything else - to express the idea of belief-strength. The rules are the same.

Other people claim that probability has a different meaning from belief-strength, or even that there is more than one meaning of the word 'probability.' But the Bayesian belief-strength theory is always applicable to tackle the problems - such as noisy parameter estimation - which they consider. Accordingly it can be handed over, changing back the word *probability* to *belief-strength* to satisfy them but leaving the content intact. The belief-strength theory is the most general known; no alternative viewpoints (*such as*

*fuzzy logic*) have done anything to extend the type of problems accessible. By contrast, the most influential non-Bayesian school, called *frequentist*, is more restricted than the Bayesian in the problems it aims to treat. We shall also suggest that it is logically flawed. (Incidentally there are differing shades of the word 'Bayesian,' and a useful test is to see whether Cox's derivation is referred to.)

In summary, probability theory is a uniquely consistent theory of logic -- *inductive* logic - used in reasoning about propositions which are either true or false, but where we do not have the information to be certain which. In particular, it is as applicable to the past as it is to the future; what, for example, is the probability, based on expert study, that a particular painter and not his pupils painted a given Old Master? *All* probabilities (belief-strengths) are conditional; statements such as "the probability that a pupil painted it is such-and-such" are incomplete without stating the evidence brought to bear - the conditioning propositions. Propositions of the type "the tree is between height  $h$  and  $h + d$ " enable quantitative and continuous parameters to be handled.

Probability theory cannot tell us which propositions or hypothesis to entertain in relation to each other. It enables us to relate propositions of our own choosing.

An important consequence of the product and sum rules, and of commutativity of the logical product, is Bayes' theorem, used in incorporating a further conditioning proposition. If we learn that proposition B is true, how does this affect our knowledge about proposition A, given at all stages the truth of proposition C? We wish to find the 'posterior' probability  $p(A|BC)$  from the 'prior' probability  $p(A|C)$ . These are related by

$$p(A|BC) = p(A|C)p(B|AC)/p(B|C).$$

In this context  $p(B|AC)$  is known as the likelihood. The quantity  $p(B|C)$  in the denominator may now be expanded using the marginalising rule, which is a consequence of the sum and product rules; Bayes' theorem is the result. The denominator is

$$p(B|C) = p(BA|C) + p(B\bar{A}|C)$$

or

$$p(B|C) = p(BA|C)p(A|C) + p(B\bar{A}|C)p(\bar{A}|C).$$

If the likelihoods  $p(BA|C)$  and  $p(B\bar{A}|C)$ , and the

prior probability  $p(A|C)$  (and hence its complement,  $p(\bar{A}|C)$ ), are known, the posterior probability  $p(A|BC)$  can be worked out. It is guaranteed from the construction of the sum and product rules that the result of incorporating two or more items is dependent -- as it must be -- only on their joint logical product and not on their order of incorporation. Bayes' theorem is applicable in estimating physical parameters from noisy measurements where the statistics of the noise are given.

Bayesian updating is also used in *inverse reasoning*, relating the propositions A and B. For example, if we are in an unfamiliar region (proposition C) and we find flourishing a plant which is known to prefer a humid climate, we tend to believe the local climate is humid. Here, A is "the climate is humid" and B is "the plant grows well"; it is readily shown that  $p(A|BC) > p(A|C)$  if  $p(B|AC) > p(B|\bar{A}C)$ . Evidently, the rules of probability encapsulate much that is intuitive; the aim of probability theory is to make this precise.

The main non-Bayesian view of probability, dominant for a century and still common today, holds that it is proportion, or relative frequency; the probability of heads in coin tossing is taken to be the ratio of heads to the number of tosses, as this number increases without bound. This is the *frequentist* view. Its supporters point out (correctly) that proportions, on a Venn diagram, satisfy the sum and product rules. But trials with coins, dice and so on need to be 'random', and this is not a clearly defined concept. On closer examination, what writers mean by random is *unpredictable*; but by whom? A table of 'random' numbers (or, in partial recognition of the problem, "pseudo-random numbers") is not unpredictable by somebody who knows about the algorithm used to generate it; nor is the landing of a die to somebody who has a timed sequence of photographs of it in flight. Randomness, or stochasticity, should not be seen as something that is intrinsic to a table of numbers or a physical system; prediction depends on the information one has about these - as Bayesians recognize. Like *subjective* and *objective* (and perhaps even *probability*), the word *random* is so laden with potential misunderstanding that it is better avoided.

There are further problems with the frequentist view of probability. In practice one never has an infinity of trials, and the finite number recorded comprise, in a big enough space, a single event. In order to cope with one-off events, frequentists invent imaginary

ensembles of alternative outcomes which might have happened (but didn't) in a 'random' trial, and use a terminology which suggests that these outcomes are real; that a parameter whose value is fixed (but unknown) actually takes many values! Frequentists also accept prior information only in the form of proportions; yet, you would not trust a doctor who stated that you should have an operation because 70% of patients showing your symptom needed it, but who refused to look at your personal medical file.

These criticisms stand alone. But, by adopting the Bayesian viewpoint, we see the trouble more clearly. Proportion is a physical, measurable quantity; its value can be used as conditioning information in assigning a probability, but it is not itself a probability. Applied in this way to repeated trials, the numerical value of probability often coincides with proportion; but probability remains a logical, and not a measurable, quantity. Bayesians also recognize and use non-proportion prior information.

Because frequentists reject the use of non-proportion prior information, for parameter estimation from logically independent noisy samples they use not Bayes' theorem, but a variety of *ad hoc* techniques known collectively as sampling theory. (Logical independence means that knowledge of one sample value does not affect our knowledge of any other.) In estimating a parameter  $\theta$  from the sample values  $\theta_1, \theta_2, \dots$ , and conditioning propositions I, sampling theory sets up an 'estimator'  $\theta^*(\theta_1, \theta_2, \dots)$  and takes the probability density  $P(\theta|\theta^*, I)$  proportional to  $P(\theta|\theta_1, \theta_2, \dots, I)$ . Even if the estimator is a 'sufficient statistic' (meaning that the data enter the likelihood  $P(\theta_1, \theta_2, \dots, \theta, I)$ , which is equal from the product rule to  $P(\theta_1, \theta_2, \dots, \theta, I)P(\theta|\theta_1, \theta_2, \dots, I)$ ..., only as a function of  $\theta^*$ ), this procedure is in violation of Bayes' theorem, unless the prior probability density for  $\theta$  is uniform. If the estimator is not sufficient the violation is worse. We see here the twin problems of sampling theory: failure to incorporate prior information, and inequivalence of the method to the sum and product rules. These account for the preference of one estimator over another in particular problems (and for many 'paradoxes' in probability theory.) The divide between 'statistics' and statistical *inference* is also seen as an artificial one.

Turn now to the rules for assigning probability in the case of a quantitative variable, which for simplicity we take at this stage as discrete. Its values each have unit weighting ('degeneracy') and are labelled by  $\{i\}$ . The crucial idea is the information content of a *distribution*: a

sharply peaked *distribution* for a parameter is clearly more informative than a broad one, and the limit in which all the probability is heaped on one value corresponds to certainty about the value of the parameter. Supposing that we hold the expression for the information content of a distribution, probability assignment proceeds by selecting that particular distribution, out of those consistent with whatever constraints are given (such as the mean or the variance), having the least information content. This distributes the probability as widely as possible. Operationally, it is a routine exercise in the variational calculus. The idea is that to choose a more informative distribution is to pretend to information we do not possess, and this somewhat abstract rationale in fact fulfills the intent of codifying and extending our intuition.

It remains to characterize the information content of a distribution  $p_i$ . This was done by Claude Shannon, who took information content to be the complement of the expected number of possibility-halving questions whose answers take us, from our current state of knowledge, to one of certainty in which the probability is all placed on a single possibility. It is conventional to work directly with this expected number of questions, which is known as the information entropy,  $S$ , and takes the famous form

$$S[p_i] = - \sum_i p_i \log_2(p_i).$$

Shannon's expression follows, from its interpretation, by partitioning the space of outcomes into subspaces, each carrying probability equal to the sum of probabilities of the elements within; renormalizing the individual distribution in each subspace; and demanding that the information entropy of the original distribution equal that of the distribution of the subspaces, plus the information entropy of each subdistribution weighted by the probability of its subspace. Only this recipe is independent of the partitioning, which can be performed in many ways. Again the criterion is one of consistency, and again it leads to a functional equation, whose solution is Shannon's form. Minimizing the information content in order to assign a distribution is equivalent to maximizing its complement, the information entropy - hence the *principle of maximum entropy*. The logarithmic form ensures non-negativity of the resulting probabilities. The information entropy is a

**Second Annual Meeting of ISBA**

At the First Meeting of the International Society for Bayesian analysis in San Francisco, the membership

minimum, of value zero, for the 'certainty'

distribution, and is a maximum for the uniform distribution in which equal probability is placed on each outcome. The uniform distribution - probability 1/6 for each fact of a six-sided die - is therefore the distribution assigned in the absence of constraints (other than normalization), as intuition would suggest, and it underlies all 'combinatorial' theorems of probability theory.

Continuous spaces can be dealt with as the limit of discrete ones. The result is that we are now to maximize

$$- \int dx p(x) \log[p(x)/m(x)],$$

where  $m(x)$  is the measure on the space  $\{x\}$ , the continuum analogue of degeneracy in the discrete case. In the absence of constraints, the maximum entropy distribution is the normalized measure. In *ab initio* continuum problems this distribution can often be found by symmetry arguments based on the meaning of the variables - furnishing the measure for constrained problems. For example the probability density for the angular location of a bead on a circular wire, with no further information, is invariant under translation, and this condition induces a functional equation for the density (the measure) whose solution is a constant.

Fuller details of these ideas about probability are contained in the text *Rational Descriptions, Decisions and Designs* by Myron Tribus (Pergamon, 1969).



voted to hold the Second Annual Meeting on June 10th and 11th, 1994, immediately following the Fifth Valencia Meeting. This ISBA meeting will include reports on applied and theoretical Bayesian analyses in the physical, biological and social sciences. Bayesians in astronomy, physics, geology, biology, medicine, economics, psychology, sociology, law, business, government and other areas are invited to submit papers for presentation at the ISBA Alicante meeting. The costs of attending (two complete days) are estimated to be:

Registration fee	(members):	5 000 pesetas
	(nonmembers):	10 000 pesetas
Accommodation and food	(double):	20 000 pesetas
	(single):	25 000 pesetas
Banquet(*)		5 000 pesetas

To ensure availability of the hotel at the above reduced rates we must know, **as soon as possible** if you plan to attend the ISBA meeting. If we do not receive enough preregistration by then, alternative venues would have to be explored.

(\*)The banquet will be on Thursday evening June 9, and will be joint Valencia Meeting - ISBA banquet. Those attending the Valencia Meeting need not pay extra for the banquet.

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**REGISTRATION FORM**

Second Annual Meeting of the International society for Bayesian Analysis

My probability of attending is \_\_\_\_\_

I plan to contribute a paper: Yes \_\_\_ No \_\_\_.

Name:

Affiliation:

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